力學（流動、波動）

某液體在靜止時，其表面漣漪在時間*t*與位置*x*的高度可用h(*t*,*x*)=sin(*t*–*kx*) 來描述。這裡的是角頻率，*k*是波數；當和*k*都大於0時，h(*t*,*x*)所描述的就是隨著時間增加而朝 +*x*方向傳播的行進波。不過在此我們並不限定值和*k*值的正負號。而觀察同一波峰或波谷位置隨時間的變化，即可定出波速(相速度)。

(1) 假如此液體朝-*x*方向做等速而穩定均勻的流動，流速為–v (v>0)。在跟隨流體移動的座標下 (液體相對於此座標是靜止的) 其表面漣漪的角頻率和波數仍為與*k*，而在實驗室的靜止座標下，其表面漣漪的角頻率和波數則為' 和*k'*。請將' 和*k'*表示成與*k*的函數(不考慮狹義相對論)。

Ans: In the comoving frame, h(*t*,*x*)=sin(*t* – *kx*), while

in the lab frame (rest frame),*t' = t*and*x' = x –*v *t* ,

so*x = x' +* v *t*and

h=sin[*t'* – *k*(*x' +* v *t*)] = sin[(v *k**t'* – *kx'* ] := sin(*'t'* – *k'x'*).

Thus ' = – v *k* and *k' = k*.

(2) 假如這種液體在靜止時的和*k*有如下圖所示的色散關係(*k*):

而在液體穩定流動時，其表面漣漪的色散關係在隨流體移動的座標下不變。以下請用作圖法回答 [提示：用直尺]，估計到小數點以下第一位即可：

[a] 在隨流體移動的座標下看起來，表面漣漪最大的可能波速vmax為何?

Ans: In the comoving frame, phase velocity = /*k*

= slope of the chord line joining (*k*, *k*) and (0, 0) in the above plot.

Thus vmax= 2 happens around *k*→0 [see the orange tangent line].

[b] 假如在實驗室的靜止座標下液體向–*x*方向流動的速率v = 1/2，那麼在實驗室座標下角頻率'=1/2的表面漣漪會有哪幾種可能的波長？在實驗室的靜止座標和隨流體移動的座標下，這些波各是往哪些方向行進的?

Ans: There are 3 intersections of the curve (*k*)[blue]and the straight line

' = 1/2 =  –(1/2) *k*[red]. They are, approximately,

1) (*k*, ) = (0.351767, 0.675883), whose wavelength  = 2/(0.351767) = 17.8618,

in the comoving frame /*k>*0, moving to the +*x* (right) direction,

in the lab frame '/*k >*0, moving to the +*x* (right) direction.

2) (*k*, ) = (2.97939, 1.98969), whose wavelength  = 2/(2.97939) = 2.10888,

in the comoving frame /*k >*0, moving to the +*x* (right) direction,

in the lab frame '/*k >*0, moving to the +*x* (right) direction.

3) (*k*, ) = (–4.99964,–1.99982),whose wavelength  = 2/|-4.99964| = 1.25673,

in the comoving frame /*k >*0, moving to the +*x* (right) direction,

in the lab frame '/*k<*0, moving to the –*x*(left) direction.