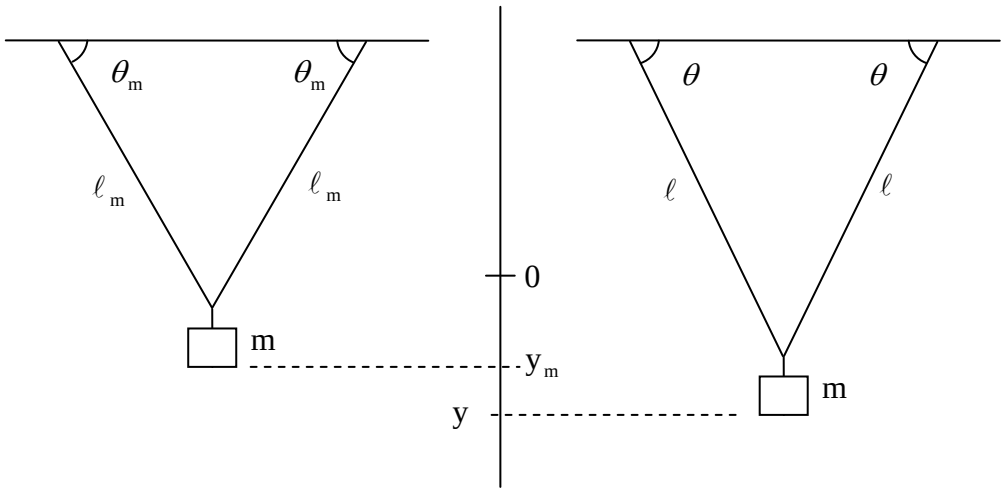
101 學年度高級中學自然學科競賽高雄市複賽

物理科筆試參考解

《第一題》



考慮鉛直方向

二彈簧懸掛質量 m，平衡時二彈簧長度各為ℓ_m，和橫樑夾角各為θ_m。

$F_y = 2k(\ell_m - \ell_0)\sin\theta_m - mg = 0$

將 m 由平衡位置向下拉一長度 Δy 到 y 點，彈簧和橫樑夾角θ

$$
\begin{aligned}
F'_y &= 2k(\ell - \ell_0)\sin\theta - mg \\
&= 2k(\ell - \ell_0)\sin\theta - 2k(\ell_m - \ell_0)\sin\theta_m \\
&\approx 2k(\ell\sin\theta - \ell_m\sin\theta_m) \\
&= 2k(y - y_m)
\end{aligned}
$$

$$\therefore \omega^2 = \frac{2k}{m}$$

《第二題》

均勻彈簧在作用力 F 下

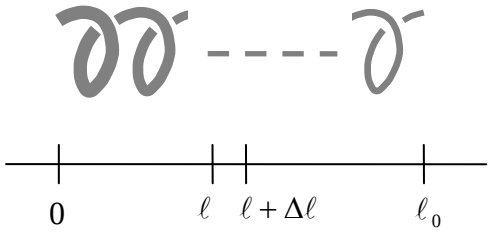
$$F = kx = k\ell_0(x/\ell_0)$$

彈簧 A 任取一段，伸長比率皆為x/ℓ_0。由楊氏公式，拉力密度$\frac{F}{A} = Y\frac{\Delta\ell}{\ell}$。

在相同作用力下，長度變化的比率和材料的截面積成反比。

若考慮切力，仍有類似的公式。切力使鋼絲有角度的變化， 造成彈簧長度變化。

鋼絲 B 製成彈簧 B 後　$a(\ell)=\dfrac{a_0}{1+\ell/\ell_0}$



彈簧 B 在 ℓ 處取 $\Delta\ell$ 長度，如右圖。

伸長比率為 $\dfrac{x}{\ell_0}(1+\dfrac{\ell}{\ell_0})$ 。

彈簧 B 在作用力 F 下，總共伸長量為

$$\int_0^{\ell_0}\frac{x}{\ell_0}(1+\frac{\ell}{\ell_0})d\ell=\frac{x}{\ell_0}(\ell_0+\ell_0/2)=\frac{3x}{2}\text{ 。}$$

故 $k_B=\dfrac{2k_0}{3}$ 。

《第三題》

(1)

From graph

$$f=\frac{1}{T}=\frac{1}{4\text{ ms}}=250\text{ Hz}\quad\#$$

$$v=\frac{0.5\text{ cm}}{0.25\text{ ms}}=20\ \text{m}/\text{s}\quad\#$$

$$\lambda=\frac{v}{f}=0.08\text{ m}=8\text{ cm}\quad\#$$

$$y(x,t)=y_m\sin[kx-\omega t+\phi]\qquad(1)$$

$$y_m=5\text{ mm}=5\times10^{-3}\text{ m}\qquad(2)$$

$$k=\frac{2\pi}{\lambda}=25\pi\text{ m}^{-1}\qquad(3)$$

$$\omega=2\pi f=500\pi\text{ s}^{-1}\qquad(4)$$

$\because y(0.005,0)=y_m$ 代入(1) with (2), (3), (4)

$$\Rightarrow\phi=\frac{3}{8}\pi$$

$$\therefore y(x,t)=0.005\sin\left[25\pi x-500\pi t+\frac{3}{8}\pi\right]\quad\text{m}\quad\#$$

with x in m, t in s.

(2)

$$y_1=y_m\sin[kx-\omega t]$$

$$y_2=y_m\sin[kx+\omega t]$$

$$y=y_1+y_2=[2y_m\sin kx]\cos\omega t$$

$\Rightarrow A = 2y_m \sin kx$ (5)

代入 $y_m = 0.005\ \text{m}$ ， $x = 0.005\ \text{m}$ ， $k = 25\pi\ \text{m}^{-1}$

$$\Rightarrow A = 0.01\sin[25\pi \times 0.005]\ \text{m}$$
$$= 0.01\sin[0.125\pi]\ \text{m}$$
$$= 0.01\sin\left[\frac{\pi}{8}\right]\ \text{m}$$

$$\because \cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\Rightarrow \sin\frac{\pi}{8} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

at $x = 0.005\ \text{m}$

$$A = 0.01 \times \frac{\sqrt{2-\sqrt{2}}}{2}\ \text{m} = 3.8268 \times 10^{-3}\ \text{m}$$

《第四題》

$$\Delta K = -\Delta U$$

$$\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 = (m_1 - m_2)gR$$

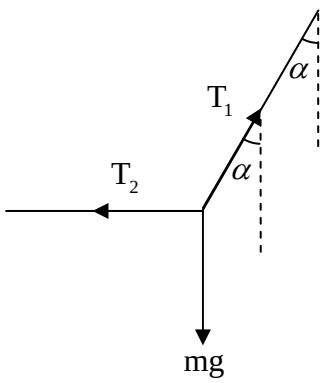
$$\frac{1}{2}m_{tot} v^2 = (0.5 - 0.2) \times g \times 0.4$$

$$v = \sqrt{\frac{2(0.5-0.2)\times g \times 0.4}{0.5+0.2}} = \sqrt{\frac{12}{35}g} \overset{\text{with } g=9.8\ \text{m/s}^2}{=} \sqrt{3.36}\ \text{m/s} = 1.83\ \text{m/s}$$

《第五題》

(1)

$$T_1 \sin\alpha = T_2$$
$$T_1 \cos\alpha = mg$$
$$\Rightarrow T_2 = mg\tan\alpha$$

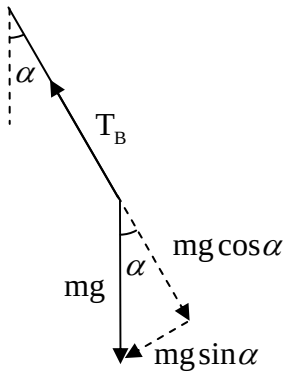


$$T_1 = \frac{T_2}{\sin\alpha} = \frac{mg\tan\alpha}{\sin\alpha} = mg\sec\alpha$$

(2)

$$mg\cos\alpha = T_B$$

$$\therefore \frac{T_B}{T_1} = \frac{mg\cos\alpha}{mg\sec\alpha} = \cos^2\alpha$$



(3)

$$mg\sin\theta = ma_t$$

$$a_t = g\sin\theta$$

$$a_r = \frac{v^2}{\ell}$$

$$\Delta K = -\Delta U$$

$$\Rightarrow \frac{1}{2}mv^2 = -mg\ell(\cos\alpha - \cos\theta)$$

$$\Rightarrow v^2 = -2g\ell(\cos\alpha - \cos\theta)$$

$$\therefore a_r = \frac{-2g\ell(\cos\alpha - \cos\theta)}{\ell} = -2g(\cos\alpha - \cos\theta)$$

$$|\vec{a}| = \sqrt{a_t^2 + a_r^2} = \sqrt{(g\sin\theta)^2 + [2g(\cos\alpha - \cos\theta)]^2}$$

$$= g\sqrt{\sin^2\theta + 4(\cos\alpha - \cos\theta)^2}$$