臺灣省中二區102學年度高級中學數理及資訊學科能力競賽

 　 物理科理論試題 解答 　 編號：

\*\* 本試題共3頁，試題請連同答案卷一併繳回 \*\*

1. 長斜坡(斜角)，球以初速(發射角)射出，如右圖。(虛線為法線、重力加速度為)。
	1. 球第一次落坡之入射角為*φ*1，求tan*φ*1 = ? (5%)
	2. 球第二次落坡之入射角為*φ*2，求tan*φ*2 = ? (3%)
	3. 球第*n*次落坡之入射角為*φ*3，求tan*φ*3 = ? (2%)

所有碰撞皆為彈性碰撞，答案以、表示

**參考答案：**













 推廣

1. 如右圖所示，有一半徑為*R* 、質量為*M* 的均勻圓盤，在光滑斜面上作純滾動，圓盤相對其轉動軸的轉動慣量為，且斜面與鉛垂線的夾角為θ。在斜面某處(設為點O)鉛直向上距離*d*的點A處是一吸引力中心，吸引力的方向是沿著點A與圓盤質量中心的連線指向點A，且其值正比於其間之距離。
2. 若在圓盤質量中心處黏貼質量為的微小質點，並將圓盤放置在此光滑斜面上，且圓盤與斜面的接觸點與點O距離為*d*，則發現圓盤會靜止不動，試求得該吸引力的比例常數*k*。(4%)
3. 若小質點突然掉落，試證明圓盤將沿著斜面進行簡諧振動(4%)，並求出
4. 圓盤沿著斜面作簡諧振動的振幅(4%)以及
5. 振動角頻率。(3%)

**參考答案：**

(a)

 

(b) 根據牛頓第二運動定律，圓盤在處受沿著斜面的合力為

 ，

滿足虎克定律，因此圓盤沿著斜面進行簡諧振動。註：為平衡點如(c)中所示。

(c)

 。

1. 圓盤沿著斜面進行簡諧振動時，可等效視之為作簡諧振動的質量-彈簧系統，其等效質量及等效力常數滿足虎克定律，結合牛頓第二運動定律得到

 ，

此時振動角頻率為 。

 當圓盤在處，其系統的總動能為，

 根據(b)可知 。因此，

1. 有一人造衛星在地球上空受重力以穩定的圓形軌道繞行，已知此一人造衛星受到地球的重力，為在地表時的0.900倍，若忽略人造衛星所受大氣層的阻力，且地球半徑為6380公里，地面之重力加速度為9.80 m/s2 ，則
	* 1. 推算此人造衛星距地表的高度為何？(3%)
		2. 此人造衛星繞行地球的週期為何？(3%)
		3. 透過太陽的反射，人們可以在黃昏時，在地面上用肉眼看見此人造衛星呈現微亮之光點，逐漸繞行。試推算此人造衛星，每繞行一周的過程，在地平線之範圍內，能被地面觀察者觀看的時間長度為何？(4%)

**參考答案：**

(a)  高度 *h* = 0.0541RE = 345 km

(b)  且  則  = 5486 sec

(c) θ= cos -1 (1/1+h) = 18.4°

地平線範圍: 2θ= 36.8°

可觀察時間: 5486×(36.8/360) = 561sec

1. 靜電學中的高斯定律是如此描述的:若考慮一淨電荷 $Q\_{encl}$ 包含在一個任意的封閉曲面內[，](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))則通過此封閉曲面的電通量等於 $\frac{1}{ε\_{o}}$ 乘以封閉曲面內的淨電荷。亦即 $∅\_{E}=\sum\_{i=1}^{i=n}\vec{E\_{i}}∙∆\vec{A\_{i}}=\frac{1}{ε\_{o}}Q\_{encl}$ 。
	1. 請根據重力與電力對空間相依關係的相似性[，](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))寫出在重力學上的高斯定律。(5%)

如右[圖](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))所[示](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))[在](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[一個](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[半徑](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)為[*R*](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))、[質](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)量為*M* 且密度均勻的[星](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))球開了一條[隧道](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))[。](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[在](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)距離[中心](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))*r*的位置[，](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)有一質量為*m*的物體[。](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))

[(](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))b[)](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[請](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[找出該物體所受到](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))的[重力](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))[。](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)(5%)

[(](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))c[)](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)若星球為[地球](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[，](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)且假設隧道內無摩擦力，請估計該物體作簡諧運動[的](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))[振盪](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)週[期](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))? (5%)

[(](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))[提示](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[：](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[地球](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)的[平均](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km))[半徑](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[為](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[6371](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)[公里](http://cn.bing.com/dict/clienttranslate?mkt=zh-CN&setLang=ZH&form=BDVEHC&q=Figure%20shows%20a%20tunnel%20in%20a%20uniform%20planet%20of%20mass%20M%20and%20radius%20R.%20At%20a%20distance%20from%20the%20center%2C%20Please%20(a)%20find%20the%20gravitational%20attractive%20force%20at%20a%20distance%20r%20from%20the%20center%3B%20(b)%20find%20the%20period%20of%20the%20oscillation%20for%20the%20earth.%0A(Hint%3A%20The%20average%20radius%20of%20earth%20is%206371%20km)" \t "_blank)，質量為5.972 × 1024 *kg*，在地球表面的重力加速度為9.780 $^{m}/\_{s^{2}}$)

**參考答案：**

(a)

$$\vec{F\_{E}}\left(r\right)=\frac{Qq}{4πε\_{o}r^{2}} \hat{r} \leftrightarrow \vec{F\_{G}}\left(r\right)=-\frac{GMm}{r^{2}} \hat{r}$$

$$\frac{1}{4πε\_{o}} \leftrightarrow -G$$

$$∅\_{E}=\sum\_{i=1}^{i=n}\vec{E\_{i}}∙∆\vec{A\_{i}}=\frac{1}{ε\_{o}}Q\_{encl} ⇔ ∅\_{G}=\sum\_{i=1}^{i=n}\vec{g\_{i}}∙∆\vec{A\_{i}}=-4πGM\_{encl}$$

or $∅\_{E}=∮\_{}^{}\vec{E}∙d\vec{A}=\frac{1}{ε\_{o}}Q\_{encl} ⇔ ∅\_{G}=∮\_{}^{}\vec{g}∙d\vec{A}=-4πGM\_{encl}$

(b)

$$g\left(4πr^{2}\right)=-4πG\left(\frac{r^{3}}{R^{3}}\right)M ⇒ g=-\frac{GM}{R^{3}}r$$

$\vec{F}=m\vec{g}=-\frac{GMm}{R^{3}}\vec{r}$

(c)

$$F\_{x}=Fsinθ=-\left(\frac{GMm}{R^{3}}r\right)\left(\frac{x}{r}\right)=-\frac{GMm}{R^{3}}x$$

$$F\_{x}=ma=m\frac{d^{2}x}{dt^{2}}=-\frac{GMm}{R^{3}}x$$

$$Let x=Asinωt$$

$$-Aω^{2}sinωt=-\frac{GM}{R^{3}}Asinωt ⇒ ω=\sqrt{\frac{GM}{R^{3}}}$$

$$T=\frac{2π}{ω}=2π\sqrt{\frac{R^{3}}{GM}}$$

At *r=R* $g=\frac{GM}{R^{2}} or G=\frac{gR^{2}}{M}$$T=2π\sqrt{\frac{R^{3}}{GM}}=2π\sqrt{\frac{R}{g}}=2π\sqrt{\frac{6371×10^{3}}{9.780}}=5071 s$

1. 今有一頻率為f0之標準音叉，小明在高塔頂上敲擊此音叉時，不慎將音叉從手中滑落而讓它垂直掉落至塔底，假設音叉在整個掉落過程中為自由落體，重力加速度為g，空氣中的聲速為v。若從音叉剛掉落的瞬間，小明按下碼錶開始計時，則在音叉掉落的過程中
	* 1. 當時間為t0時音叉恰好落至A點，請求出聲音由A點傳至塔頂所需的時間。(5%)
		2. 請推導出小明在塔頂所聽到從音叉所發出的聲音頻率f與時間t的關係。(提示：要考慮聲音從音叉傳至塔頂所需的時間)。(5%)

**參考答案：**

(a) 音叉落下之距離  

(b) 假設音叉發聲的時間為*t*0，而小明聽到聲音的時間為*t*，則

  ⇒  (只取”+”，“−“不合)

 時間為*t*0時音叉落下之速度為 

 因此小明在塔頂所聽到從音叉所發出的聲音頻率f為

 

1. 假設彈簧的彈力常數k與絕對溫度成正比。黃銅的熱膨脹係數為 1.9×10-5 /℃，木頭的熱膨脹係數為 8.0×10-6 /℃。
	* 1. 小明在夏天37℃以彈簧秤秤得體重60kg。若小明體重不變，在冬天7℃以相同彈簧秤，歸零後，秤得體重為若干kg? (3%)
		2. 若機械鐘是以彈簧的周期運動來控制快慢。試估計在夏天37℃校對好的精準時鐘，到了冬天7℃時，時鐘會走的較快或是較慢? 一天誤差約多少分鐘? (3%)
		3. 以一黃銅製的直尺量一木製桌子，在夏天37℃量得桌寬為90cm。試問到了冬天7℃時，以此黃銅尺量得桌寬將會增加或是減少? 改變多少cm? (5%)
		4. 若擺鐘是以黃銅製的單擺在重力下周期運動來控制快慢。試估計在夏天37℃校對好的精準時鐘，到了冬天7℃時，時鐘會走的較快或是較慢? 一天誤差約多少秒? (4%)

**參考答案：**

(a) 體重 w與彈簧變化量Δ*x*成正比 彈力常數 k 成反比，因此體重讀數與溫度成反比。 溫度降低，體重讀數增加。因此: 

(b) 彈簧周期與彈力常數根號成反比，因此與溫度根號成反比。溫度減小，周期變長、變慢。



(c) 對尺而言，尺的長度與讀數成反比。黃銅縮的多，因此刻度變多、變長。



(d) 單擺周期與長度根號成正比。冬天時，單擺變短，周期變短、變快。



1. 如右圖所示，一透明圓柱管折射率為n1，浸入折射率為n2之透明液體中，且n1> n2。
	* 1. 為使光束能於透明圓柱管中順利傳播，試證明光束之最大孔徑角u滿足下列關係式，

 $1+sin^{2}u=\frac{n\_{1}^{2}}{n\_{2}^{2}}$。 (5%)

* + 1. 若n2之折射率為4/3，則透明圓柱管之折射率n1最少需為多少?(只寫答案不計分) (5%)

**參考答案：**

(a)

$n\_{1}sinθc=n\_{2}$, $sinθc=\frac{n\_{2}}{n\_{1}}$

$n\_{2}sinu=n\_{1}sin\left(\frac{π}{2}-θc\right)=n\_{1}cosθc$, $sinu=\frac{n\_{1}}{n\_{2}}cosθc$

$$sin^{2}u=\left(\frac{n\_{1}}{n\_{2}}\right)^{2}cos^{2}θc=\left(\frac{n\_{1}}{n\_{2}}\right)^{2}\left(1-sin^{2}θc\right)^{2}=\left(\frac{n\_{1}}{n\_{2}}\right)^{2}-1$$

1+$sin^{2}u$=$\left(\frac{n\_{1}}{n\_{2}}\right)^{2}$

(b)$ 1+sin^{2}u=\frac{n\_{1}^{2}}{n\_{2}^{2}},$

$$ 因sin^{2}u之極大值為1，故1+sin^{2}u\leq \frac{n\_{1}^{2}}{n\_{2}^{2}}, n\_{2}=4/3,$$

$(1+1)\left(\frac{4}{3}\right)^{2}\leq n\_{1}^{2}$,

$n\_{1}\geq \frac{4}{3}\sqrt{2}$



1. (a)如圖所示，一物體置於焦距5cm的凸透鏡前方5cm處，另一焦距-15cm的凹透鏡則置放於凸透鏡後方10cm處，請問最終成像位置與放大率為何? (5%)

物體

透鏡二

透鏡一

10cm

5cm

*f*1=5cm

*f*2= -15cm

(b)承上題，若將另一焦距4cm的凸透鏡取代凹透鏡並將之置放於前一凸透鏡後方4.5cm處，請問最終成像位置與放大率為何? (5%)

透鏡一

物體

透鏡二

4.5cm

5cm

*f*1=5cm

*f*2= 4cm

(c)承上題，若將圖中焦距4cm的凸透鏡置換為焦距3cm的凸透鏡後，但仍要維持成像面位置不變(即成像面與第二片凸透鏡的相對位置不改變)，在物體與透鏡二者均不能移動的情況下我們可以藉由將第一片凸透鏡往物體方向移動來達成此要求，請問此移動量*x*為多少? (5%)

*x*

*f*2= 3cm

*f*1=5cm

4.5cm

5cm

**參考答案：**

(a) 成像於透鏡一前方5cm處，放大倍率3

(b) 成像於透鏡二後方4cm處，放大倍率0.8

(c) 2.5cm