International Journal of Bifurcation and Chaos, Vol. 18, No. 12 (2008) 3731–3736

SYNCHRONIZATION OF UNCERTAIN HYPERCHAOTIC AND CHAOTIC SYSTEMS BY ADAPTIVE CONTROL

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Received November 16, 2007; Revised February 29, 2008

This paper presents the synchronization between uncertain hyperchaotic and chaotic systems. Based on Lyapunov stability theory, an adaptive controller is derived to achieve synchronization of hyperchaotic and chaotic systems, including the case of unknown parameters in these two systems. The T.N.Č. hyperchaotic oscillator is used as the master system, and the Rössler system is used as the slave system. Numerical simulations verify these results. Additionally, the effect of noise is investigated by measuring the mean squared error (MSE) of two systems.

Keywords: Adaptive control; hyperchaotic system; synchronization.

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1. Introduction

Chaos synchronization is one of the critical issues in nonlinear science, for its various potential applications in physics, secure communications, chemical reactors, control theory, biological networks, and artificial neural networks, etc. Particularly, in recent neuroscience research, synchronization plays a very important role in the analysis of migraine and in the application of epilepsy. In the recent decades, the synchronization of chaotic systems has been extensively investigated both theoretically and experimentally [Pecora & Carrol, 1990; Kocarev & Parlitz, 1995; Parlitz *et al.*, 1996; Fischer *et al.*, 2000; Wang *et al.*, 2002; Fotsin *et al.*, 2006]. Many researchers had shown the possibility to achieve synchronization between different chaotic systems [Agiza & Yassen, 2001; Ho *et al.*, 2002; Ho & Hung, 2002; Ho *et al.*, 2005; Yassen, 2005; Wu *et al.*, 2007]. However, there are few discussions in scientific literature about synchronization between hyperchaotic and chaotic systems. Notably, hyperchaotic systems have been extensively investigated [Peng *et al.*, 1996; Duan & Yang, 1997; Grassi & Mascolo, 1998; Grassi & Mascolo, 1999; Miller & Grassi, 2001; Udaltsov *et al.*, 2001; Li *et al.*, 2005; Yan & Yu, 2007]. When a system has more than one positive Lyapunov exponent, it can clearly generate more complicated dynamics. The complex behaviors of hyperchaotic systems are believed to have much wider applications. However, most of the methods are valid only when the parameters of the systems are known. In practice, some or all of the parameters of the systems cannot be exactly known *a priori*. Therefore, the synchronization of two uncertain chaotic systems is essential.

In this paper, adaptive control theory [Liao, 1998; Chen & Lü, 2002; Wang *et al.*, 2004; Park, 2005; Yassen, 2005; Ho *et al.*, 2006; Li *et al.*, 2007] is applied to achieve the synchronization between uncertain chaotic and hyperchaotic systems. These techniques were successfully applied to the Rössler system and the Tamaševičius, Namajūas, and Čenys (T.N.Č.) hyperchaotic systems [Tamaševičius *et al.*, 1996]. This paper is organized as follows. In Secs. 2 and 3, the Rossler system is controlled and synchronized with T.N.Č. system under some known and some unknown parameters, respectively. Section 4 discusses the mean squared error (MSE) between the two synchronized systems. Finally, conclusions are drawn in Sec. 5.

2. Controlling the Rössler System to Synchronize with the T.N.Č. System Under the Determinate Parameters

Consider a very simple hyperchaotic oscillator as a drive system introduced by [Tamaševičius *et al.*, 1996]. The oscillator consists of a combined parallel-series LC circuit, a single op amp, a negative resistance and a diode as a nonlinear device. They are hyperchaotic oscillators with certain assigned parameter values. The equations of the driving system are as follows:

$$\begin{aligned}\dot{x}_1 &= a_1x_1 - x_2 - x_3,\\ \dot{x}_2 &= x_1,\\ \dot{x}_3 &= c_1(x_1 - x_4),\\ \dot{x}_4 &= d_1[x_3 - b_1(x_4 - 1)H(x_4 - 1)],\end{aligned} \tag{1}$$

where $H(u)$ is the Heaviside function, with the definition $H(u < 0) = 0$ and $H(u \geq 0) = 1$. The response system is [Rössler, 1976] system:

$$\begin{aligned}\dot{y}_1 &= -y_2 - y_3 + u_1,\\ \dot{y}_2 &= y_1 + a_2y_2 + u_2,\\ \dot{y}_3 &= b_2 - c_2y_3 + y_1y_3 + u_3.\end{aligned} \tag{2}$$

Let all of the parameters of these two systems be known, where $U = (u_1, u_2, u_3)^T$ is the controller to be determined. Since the orders of these two systems are different, we set partial dimensions of the driving system as the goal to reach synchronization. Accordingly, Eq. (1) is subtracted from Eq. (2) and the following notations are applied.

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3, \tag{3}$$

yielding,

$$\begin{aligned}\dot{e}_1 &= -e_2 - e_3 - a_1x_1 + u_1,\\ \dot{e}_2 &= e_1 + a_2e_2 + a_2x_2 + u_2,\\ \dot{e}_3 &= x_3e_1 + (x_1 - c_1)e_3 + e_1e_3 - c_1(x_1 - x_4)\\ &\quad - c_2x_3 + x_1x_3 + b_2 + u_3.\end{aligned} \tag{4}$$

The Lyapunov function of Eq. (4) is selected as follows

$$V(\mathbf{e}) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) > 0 \tag{5}$$

where $\mathbf{e} = (e_1, e_2, e_3)^T$ and controller U is designated as follows

$$\begin{aligned}u_1 &= -k_1e_1 - x_3e_3 - e_3^2 + a_1x_1,\\ u_2 &= -(k_2 + a_2)e_2 - a_2x_2,\end{aligned}$$

$$\begin{aligned} u_3 = &-k_3e_3 + e_1 - (x_1 - c_2)e_3 \\ &+ c_1(x_1 - x_4)c_2x_3 - x_1x_3 - b_2. \end{aligned} \tag{6}$$

where $\forall\, k_i > 0, i = 1, 2, 3$; then the derivative along the trajectory of Eq. (4) is

$$\begin{aligned} \frac{dV(\mathbf{e})}{dt} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(-e_2 - e_3 - a_1x_1 + u_1) \\ &\quad + e_2(e_1 + a_2e_2 + a_2x_2 + u_2) \\ &\quad + e_3(x_3e_1 + (x_1 - c_2)e_3 + e_1e_3 \\ &\quad - c_1(x_1 - x_4) - c_2x_3 + x_1x_3 + b_2 + u_3) \\ &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 < 0. \end{aligned}$$

Based on Lyapunov stability theory, the error dynamical system Eq. (4) is globally and asymptotically stable, i.e.

$$\lim_{t\to\infty} \|e_i\| = \lim_{t\to\infty} \|y_i - x_i\| = 0, \quad i = 1, 2, 3.$$

Therefore, the states y_1, y_2 and y_3 of the response system will be synchronized with the corresponding states x_1, x_2 and x_3 of the driving system.

The effectiveness of the proposed synchronization approach is numerically simulated. The parameters of the T.N.Č. hyperchaotic system are selected as $a_1 = 0.7$, $b_1 = 10.0$, $c_1 = 3.0$, $d_1 = 3.0$ and the three parameters of the Rössler system are $a_2 = 0.2$, $b_2 = 0.2$, $c_2 = 5.7$.

According to Eq. (6), the controller (u_1, u_2, u_3) can be determined as $(-k_1e_1 - x_3e_3 - e_3^2 + 0.7x_1, -(k_2 + 0.2)e_2 - 0.2x_2, -k_3e_3 + e_1 - (x_1 - 5.7)e_3 + 3(x_1 - x_4) + 5.7x_3 - x_1x_3 - 0.2)$. The initial conditions of the driving and response systems are $(0.4, 0.1, 0.2, 0.3)$ and $(0.1, 0.3, 0.5)$ respectively. Setting $k_1 = k_2 = k_3 = 1$ and a time step of 0.001, we start to control the system after $t \geq 200$. Figure 1 presents the trajectories of e_1, e_2 and e_3, as it can be readily seen that the error dynamical system tends to zero after control.

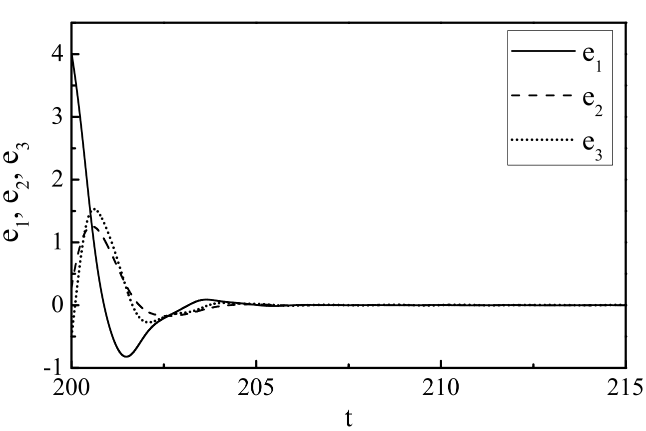

Fig. 1. Synchronization error of the Rössler and T.N.Č. hyperchaotic systems when all parameters are known. The control is initiated at $t = 200$. Solid line: $e_1 = y_1 - x_1$, dash line: $e_2 = y_2 - x_2$, dot line: $e_3 = y_3 - x_3$.

3. Controlling the Rössler System to Synchronize with the T.N.Č. System Under the Uncertain Parameters

This section will discuss the situation in which all of the parameters of these two systems are unknown. Assume that a_1, b_1, c_1, d_1, a_2, b_2, c_2 are some unknown constant parameters, and $U = (u_1, u_2, u_3)^T$ is the controller to be designated. Following the same way described in the previous section, Eq. (1) is subtracted from Eq. (2) and we denote

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3, \tag{7}$$

yielding,

$$\begin{aligned} \dot{e}_1 &= -e_2 - e_3 - a_1x_1 + u_1, \\ \dot{e}_2 &= e_1 + a_2e_2 + a_2x_2 + u_2, \\ \dot{e}_3 &= x_3e_1 + (x_1 - c_1)e_3 + e_1e_3 - c_1(x_1 - x_4) \\ &\quad - c_2x_3 + x_1x_3 + b_2 + u_3. \end{aligned} \tag{8}$$

Now, we select the Lyapunov function of Eq. (8) as,

$$\begin{aligned} &V(\mathbf{e}, \tilde{a}_1, \tilde{c}_1, \tilde{a}_2, \tilde{b}_2, \tilde{c}_2) \\ &\quad = \frac{1}{2}(\mathbf{e}^T\mathbf{e} + \tilde{a}_1^2 + \tilde{c}_1^2 + \tilde{a}_2^2 + \tilde{b}_2^2 + \tilde{c}_2^2) > 0, \end{aligned} \tag{9}$$

where $\mathbf{e} = (e_1, e_2, e_3)^T$, $\tilde{a}_1 = a_1 - \hat{a}_1$, $\tilde{c}_1 = c_1 - \hat{c}_1$, $\tilde{a}_2 = a_2 - \hat{a}_2$, $\tilde{b}_2 = b_2 - \hat{b}_2$, $\tilde{c}_2 = c_2 - \hat{c}_2$, and $\hat{a}_1$, $\hat{c}_1$, $\hat{a}_2$, $\hat{b}_2$, $\hat{c}_2$ are estimate values of the unknown parameters a_1, c_1, a_2, b_2, c_2, respectively. With suitably controlled U and the following estimates of parameters update laws $\dot{\hat{a}}_1$, $\dot{\hat{c}}_1$, $\dot{\hat{a}}_2$, $\dot{\hat{b}}_2$ and $\dot{\hat{c}}_2$, we have

$$\begin{aligned} u_1 &= -k_1e_1 - x_3e_3 - e_3^2 + \hat{a}_1x_1, \\ u_2 &= -(k_2 + \hat{a}_2)e_2 - \hat{a}_2x_2, \\ u_3 &= -k_3e_3 + e_1 - (x_1 - \hat{c}_2)e_3 + \hat{c}_1(x_1 - x_4) \\ &\quad + \hat{c}_2x_3 - x_1x_3 - \hat{b}_2, \end{aligned}$$

$$
\begin{aligned}
\dot{\hat{a}}_1 &= -x_1 e_1, \\
\dot{\hat{c}}_1 &= -(x_1 - x_4) e_3, \\
\dot{\hat{a}}_2 &= (e_2 + x_2) e_2, \\
\dot{\hat{b}}_2 &= e_3, \\
\dot{\hat{c}}_2 &= -(e_3 + x_3) e_3,
\end{aligned} \tag{10}
$$

where $\forall\, k_i > 0, i = 1, 2, 3$, and the derivative of along trajectories of Eq. (8) is

$$
\begin{aligned}
&\frac{dV(\mathbf{e}, \tilde{a}_1, \tilde{c}_1, \tilde{a}_2, \tilde{b}_2, \tilde{c}_2)}{dt} \\
&\quad = \mathbf{e}^T \dot{\mathbf{e}} + \tilde{a}_1 \dot{\tilde{a}}_1 + \tilde{c}_1 \dot{\tilde{c}}_1 + \tilde{a}_2 \dot{\tilde{a}}_2 + \tilde{b}_2 \dot{\tilde{b}}_2 + \tilde{c}_2 \dot{\tilde{c}}_2 \\
&\quad = e_1(-e_2 - e_3 - a_1 x_1 + u_1) \\
&\qquad + e_2(e_1 + a_2 e_2 + a_2 x_2 + u_2) \\
&\qquad + e_3(x_3 e_1 + (x_1 - c_2) e_3 + e_1 e_3 \\
&\qquad - c_1(x_1 - x_4) - c_2 x_3 + x_1 x_3 + b_2 + u_3) \\
&\qquad + \tilde{a}_1 \dot{\tilde{a}}_1 + \tilde{c}_1 \dot{\tilde{c}}_1 + \tilde{a}_2 \dot{\tilde{a}}_2 + \tilde{b}_2 \dot{\tilde{b}}_2 + \tilde{c}_2 \dot{\tilde{c}}_2 \\
&\quad = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - \tilde{a}_1 x_1 e_1 \\
&\qquad + \tilde{a}_2 (e_2 + x_2) e_2 - \tilde{c}_1 (x_1 - x_4) e_3 \\
&\qquad - \tilde{c}_2 (e_3 + x_3) e_3 + \tilde{b}_2 e_3 - \tilde{a}_1 \dot{\hat{a}}_1 \\
&\qquad - \tilde{b}_1 \dot{\hat{b}}_1 - \tilde{a}_2 \dot{\hat{a}}_2 - \tilde{b}_2 \dot{\hat{b}}_2 - \tilde{c}_2 \dot{\hat{c}}_2 \\
&\quad = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 < 0.
\end{aligned} \tag{11}
$$

Again, based on Lyapunov stability theory, the error dynamical system Eq. (8) is globally asymptotically stable, i.e.

$$\lim_{t\to\infty} \|e_i\| = \lim_{t\to\infty} \|y_i - x_i\| = 0, \quad i = 1, 2, 3.$$

Therefore, the states y_1, y_2 and y_3 of the response system will also be synchronized with the states x_1, x_2 and x_3 of the driving system.

A numerical simulation represents the effectiveness of the proposed synchronization approach. The unknown parameters of the T.N.Č. hyperchaotic system are set to $a_1 = 0.7$, $b_1 = 10.0$, $c_1 = 3.0$, $d_1 = 3.0$ and the three unknown parameters of the Rössler system are set to $a_2 = 0.2$, $b_2 = 0.2$, $c_2 = 5.7$. The initial conditions of the driving and response systems are (0.4, 0.1, 0.2, 0.3) and (0.1, 0.3, 0.5). Choosing $k_1 = k_2 = k_3 = 1$ and using a time step of 0.001, the system is controlled after $t \geq 200$ and the initial values of the parameters $\dot{\hat{a}}_1$, $\dot{\hat{c}}_1$, $\dot{\hat{a}}_2$, $\dot{\hat{b}}_2$ and $\dot{\hat{c}}_2$ are all set to zero. Figure 2 shows the trajectories of e_1, e_2 and e_3, and as indicated, the error dynamical system tended to zero after control.

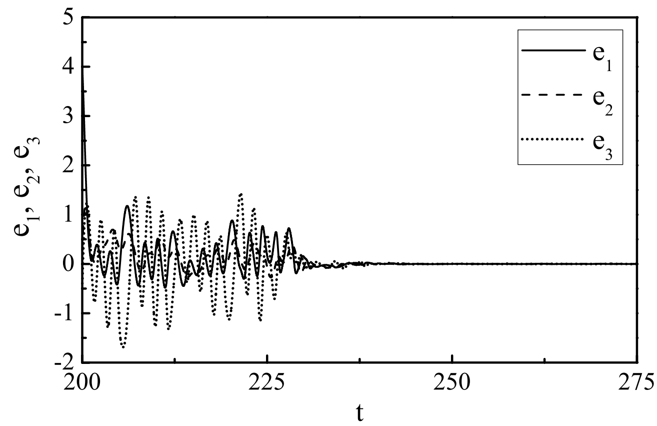

Fig. 2. Synchronization error of the Rössler and T.N.Č. hyperchaotic systems when all parameters are unknown. The control is initiated at $t = 200$. Solid line: $e_1 = y_1 - x_1$, dash line: $e_2 = y_2 - x_2$, dot line: $e_3 = y_3 - x_3$.

4. Effect of *k* and External Noise

Mean squared error (MSE) is now employed to measure the synchronized efficiency between the driver system (x_1, x_2, x_3) and the response system (y_1, y_2, y_3). It is defined as,

$$\mathrm{MSE} = \frac{1}{T} \int_0^T [\mathbf{x}(t) - \mathbf{y}(t)]^2\, dt. \tag{12}$$

From Eqs. (10) and (11), controller U has several forms. Various controllers of U are obtained by modulating values of k. For simplicity here, we assume $k_1 = k_2 = k_3 = \mathbf{k}$ and $\mathbf{k}$ varies from one to 200 in one step.

The evolution time T is set sufficiently large after the response system has been synchronized with the driver system. Figure 3 displays the simulation results. As $\mathbf{k}$ increases, the quantity of mean squared error between these two systems declines indicating that synchronization is improved.

Moreover, the influence of external noise is also examined. In real systems, the measurement noise and dynamical noise are inevitable. Accordingly, the noise analysis is essential to ensure the stability of the synchronization between these two systems. The noise is added to the driving system and Eq. (1), and can be rewritten as,

$$
\begin{aligned}
\dot{x}_1 &= a_1 x_1 - x_2 - x_3 + \xi(t), \\
\dot{x}_2 &= x_1, \\
\dot{x}_3 &= c_1(x_1 - x_4), \\
\dot{x}_4 &= d_1[x_3 - b_1(x_4 - 1) H(x_4 - 1)],
\end{aligned} \tag{13}
$$

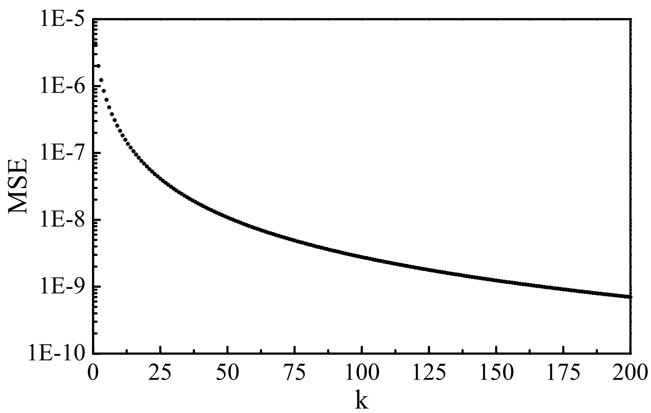

Fig. 3. The diagram presents the values of MSE versus the value **k**, where **k** varies from 1 to 200 with step 1.

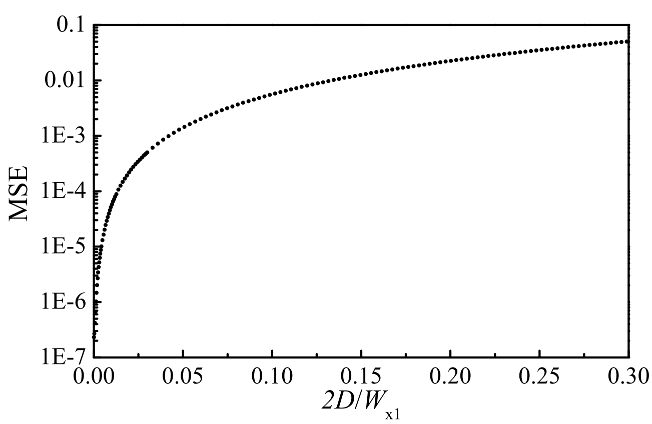

Fig. 4. The diagram presents the values of average mean squared error versus $2D/W_{x1}$ ratio.

where $\xi(t)$ is the Gaussian white noise, with $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t')$ where D is the noise intensity. The parameters of these two systems are the same as the previous section. Figure 4 shows the results of our numerical simulation, where W_{x1} represents the range of the state x_1. We set $k_1 = k_2 = k_3 = \mathbf{k} = 10$ and T sufficiently large enough to eliminate the transition state. Various noise intensities are adopted in the analysis, MSE is calculated on an average of 100 times. If we choose the variable $R_a = 2D/W_{x1}$, the relationship between MSE and R_a can be characterized as

$$\text{MSE} = \alpha + \beta R_a^{\gamma}, \tag{14}$$

where $\alpha = 2.6854E - 7$, $\beta = 0.56616$, $\gamma = 2.00301$. This simulation shows the robustness of these systems against noise.

5. Conclusion

In this paper, chaos synchronization between the hyperchaotic system and the chaotic system with all parameters unknown is presented by using the adaptive control technique. The MSE analysis reveals that the k values of the controller U, can be adjusted to yield excellent synchronization. Furthermore, external noise was added to the driving system to investigate the stability of synchronization. The result of MSE analysis demonstrate its robustness against the system noise.

Secure communication has been one of the important applications of chaotic synchronization since the last decades [Cuomo & Oppenheim, 1993]. Due to their unpredictability and broad band spectrum, chaotic signals have been used to encode information by simple masking (addition) or using modulation. Recently, the synchronization of chaotic systems with different order has been applied to such a field [Samuel, 2004]. Moreover, the use of hyperchaotic systems is able to increase the complexity of transmitting signals [Peng *et al.*, 1996]. Therefore, the synchronization scheme we propose in this paper would be beneficial in the application of chaos in secure communication.

Acknowledgments

The authors would like to thank the National Science Council of the Republic of China, Taiwan, for financial support under the contract No. NSC 96-2122-M-017-001-MY3.

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