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Synchronization of two different systems by using generalized active control

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Abstract

We have already generalized the techniques from active control theory, and applied them to synchronize two different systems. In this Letter, we demonstrate these techniques by period-system, Lorenz and Rossler systems. Moreover, the effect of external noise is also included in our discussion.

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Keywords: Active control; Synchronization; Noise

1. Introduction

Chaos synchronization is an important topic in the nonlinear science. Generally speaking, the synchronization phenomenon has the following feature: the trajectories of two systems (master and slave systems) are identical notwithstanding starting from different initial conditions. However, slight errors of initial conditions, for chaos dynamical systems, will lead to completely different trajectories. Therefore, how to control two chaos systems to be synchronized has received a great deal of interest in the past decades.

From the paper written by Pecora and Carroll [1], many other papers about controlling systems to be synchronization have been published [2–6]. Recently, Bai and Lonngren used active control techniques to synchronize two Lorenz systems [7]. Moreover, researchers examined these techniques further and applied them in other systems [8–10]. Lately, we used these techniques to achieve phase and anti-phase synchronization, and analyzed the effect of eigenvalues [11]. Based on these studies, people can easily synchronize two identical chaos systems (i.e., two chaos systems whose parameter are equal) with active control.

In this Letter, we generalize the skills of active control. In other words, we break the limit of "controlling two identical systems" and apply these techniques to synchronize two different systems. We demonstrate generalized active control by period-systems, Lorenz and Rossler systems. Moreover, the effect of external noise is also included in our discussion.

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2. Control Rossler system to be period-system

First, take an easy period-system and Rossler system into consideration.

Master system ($\dddot{x} + \dot{x} = 0$):

$$\dot{x}_1 = y_1,$$
$$\dot{y}_1 = z_1,$$
$$\dot{z}_1 = -y_1. \tag{1}$$

Slave system:

$$\dot{x}_2 = -y_2 - z_2 + u_a,$$
$$\dot{y}_2 = x_2 + ay_2 + u_b,$$
$$\dot{z}_2 = b + z_2(x_2 - c) + u_c. \tag{2}$$

Using active control techniques, we subtract (1) from (2) and get

$$\dot{x}_3 = -y_2 - y_1 - z_2 + u_a,$$
$$\dot{y}_3 = x_2 + ay_2 - z_1 + u_b,$$
$$\dot{z}_3 = b + z_2(x_2 - c) + y_1 + u_c. \tag{3}$$

Referring to the original methods of active control, we must redefine the three control functions $u_a(t)$, $u_b(t)$ and $u_c(t)$ to eliminate all items that cannot be shown in the form of x_3, y_3 and z_3. After this process, a constant matrix A will be chosen, and the control functions can be decided [7]. However, we fail to find x_3, y_3 or z_3 in Eq. (3). Thus, what we have to do is to create x_3, y_3 and z_3:

$$\begin{aligned}\dot{x}_3 &= -(y_2 - y_1) - 2y_1 - (z_2 - z_1) - z_1 + u_a \\ &= -y_3 - z_3 + u_a - 2y_1 - z_1,\end{aligned} \tag{4}$$

$$\begin{aligned}\dot{y}_3 &= (x_2 - x_1) + x_1 + a(y_2 - y_1) \\ &\quad + ay_1 + (z_2 - z_1) - z_2 + u_b \\ &= x_3 + ay_3 + z_3 + u_b + x_1 + ay_1 - z_2,\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{z}_3 &= b + z_2x_2 - c(z_2 - z_1) - cz_1 \\ &\quad - (y_2 - y_1) + y_2 + u_c \\ &= -cz_3 - y_3 + u_c + b + z_2x_2 - cz_1 + y_2.\end{aligned} \tag{6}$$

According to the clue of active control, we can use (u_a, u_b, u_c) to obviate the items which are not x_3, y_3 or z_3. By this way, the functions (u_a, u_b, u_c) can be determined:

$$u_a = v_a + 2y_1 + z_1,$$
$$u_b = v_b - x_1 - ay_1 + z_2,$$
$$u_c = v_c - b - z_2x_2 + cz_1 - y_2. \tag{7}$$

And (3) will be rewritten as:

$$\dot{x}_3 = y_3 - z_3 + v_a,$$
$$\dot{y}_3 = x_3 + ay_3 + z_3 + v_b,$$
$$\dot{z}_3 = -cz_3 - y_3 + v_c. \tag{8}$$

Finally, all we have to do is to produce a matrix A. Choosing three eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ as $(-1, -1, -1)$, A becomes:

$$A = \begin{pmatrix} -1 & -1 & 1 \\ -1 & -a-1 & -1 \\ 0 & 1 & c-1 \end{pmatrix}. \tag{9}$$

Then we get three control functions.

2.1. Numerical results

RK4 method is used to all of our simulations with time step being equal to 0.001. We select the parameters of Rossler systems as $a = 0.2$, $b = 0.2$, $c = 5.0$ to ensure the chaotic behavior. The initial values are $x_1(0) = 0.0$, $y_1(0) = 1.0$, $z_1(0) = 1.0$ and $x_2(0) = 0.2$, $y_2(0) = 0.6$, $z_2(0) = 1.0$. And control inputs start at $t = 50$.

The simulation results are illustrated in Fig. 1. Fig. 1(a) displays the signals x_1 and x_2, while Fig. 1(b) displays y_1 and y_2, Fig. 1(c) displays z_1 and z_2. As we expect, people can observe that slave system starts to trace master system and finally becomes the same after $t \geqslant 50$.

What deserves to be mentioned is values of the three eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ play an important role. When all eigenvalues are smaller than zero, the system will be convergence. If we make them get smaller, the rate of convergence will become faster and the error signals will be closer to zero. Similarly, we also can choose one of them being equal to zero, and the responding dimension will turn out to be phase-synchronization [11]. Fig. 2 presents the effect of three eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$. When we choose $\lambda_1 = \lambda_2 = \lambda_3 = -10$ (solid line), the error signals x_3 will evolve to zero rapidly. With the increase of eigenvalues ($\lambda_1 = \lambda_2 = \lambda_3 = -1$: long-dash line, and $\lambda_1 = \lambda_2 =$

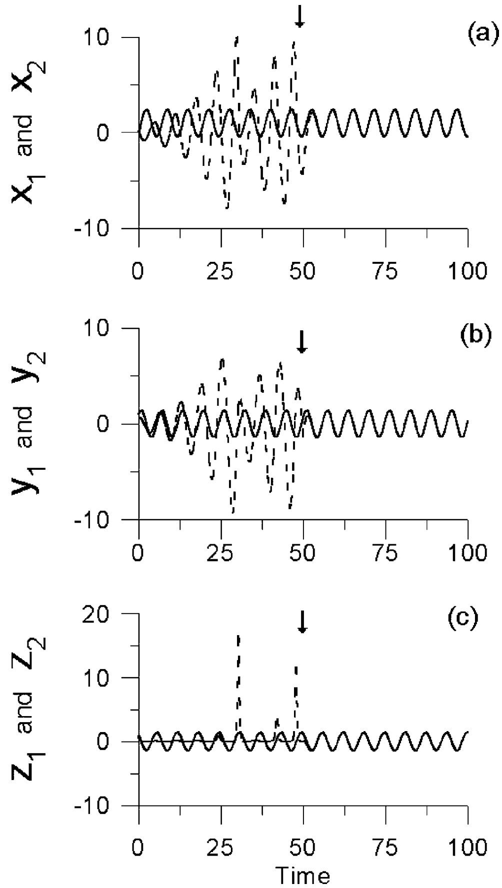

Fig. 1. The diagram of the Rossler system controlled to be period-system by using generalized active control; (a) shows the time series of signals x_1 and x_2, (b) shows the signal y_1 and y_2, (c) shows the signal z_1 and z_2. The arrows indicate the time we begin to control (period-system: solid line, Rossler system: dash line).

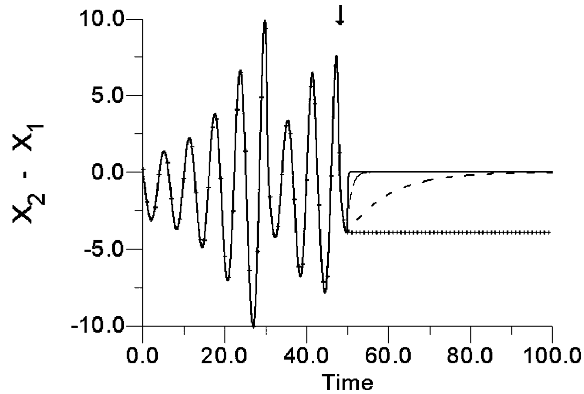

Fig. 2. Taking the same example used in Fig. 1, we choose $\lambda_1 = \lambda_2 = \lambda_3 = -10$ (solid line), $\lambda_1 = \lambda_2 = \lambda_3 = -1$ (long-dash line), $\lambda_1 = \lambda_2 = \lambda_3 = -0.1$ (short-dash line) and $\lambda_1 = \lambda_2 = \lambda_3 = 0$ (cross points) in turns to sketch the error signal $x_2 - x_1$. The arrow indicates the time we begin to control.

$\lambda_3 = -0.1$: short-dash line), it takes more time for two systems to become synchronization. Finally, when $\lambda_1 = \lambda_2 = \lambda_3 = 0$ (cross points), the signal $x_3(t)$ $(= x_2 - x_1)$ maintains a constant after $t \geqslant 50$.

3. Control two different chaos systems

Now, we want to control two different chaos systems. Take Lorenz and Rossler systems into consideration.

Master system (Lorenz):

$$\dot{x}_1 = \sigma_1(y_1 - x_1),$$
$$\dot{y}_1 = \gamma_1 x_1 - y_1 - x_1 z_1,$$
$$\dot{z}_1 = x_1 y_1 - b_1 z_1. \tag{10}$$

Slave system (Rossler):

$$\dot{x}_2 = -y_2 - z_2 + u_a,$$
$$\dot{y}_2 = x_2 + a_2 y_2 + u_b,$$
$$\dot{z}_2 = b_2 + z_2(x_2 - c_2) + u_c. \tag{11}$$

Similarly, subtract (10) from (11) and get

$$u_a = v_a - \sigma_1 x_1 + (\sigma_1 + 1) y_1 + z_1,$$
$$u_b = v_b - x_1 z_1 + (\gamma_1 - 1) x_1 - (a_2 + 1) y_1,$$
$$u_c = v_c - b_2 - z_2 x_2 + x_1 y_1 + (c_2 - b_1) z_1, \tag{12}$$

$$\dot{x}_3 = -y_3 - z_3 + v_a,$$
$$\dot{y}_3 = x_3 + a_2 y_3 + v_b,$$
$$\dot{z}_3 = -c_2 z_3 + v_c. \tag{13}$$

Select $(\lambda_1, \lambda_2, \lambda_3)$ as $(-1, -1, -1)$, then we can get:

$$v_x = z_3 + y_3 - x_3,$$
$$v_y = -x_3 - (a_2 + 1) y_3,$$
$$v_z = (c_2 - 1) z_3. \tag{14}$$

Control functions can be determined.

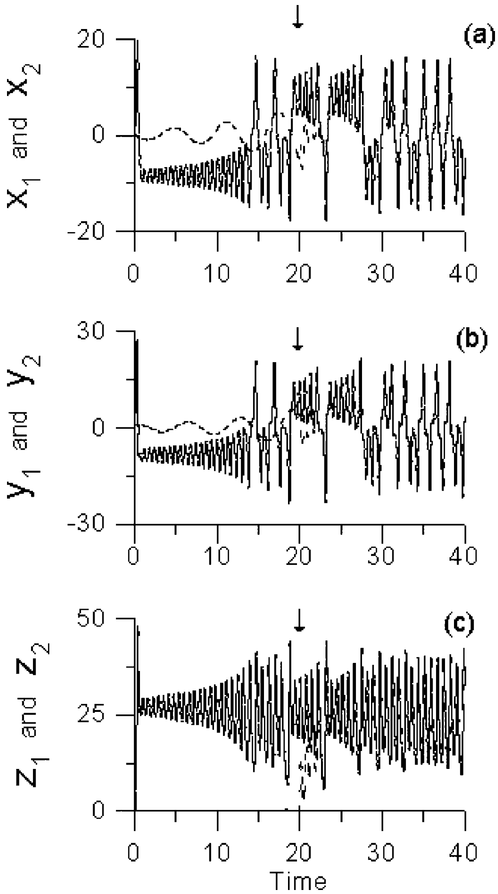

Fig. 3. The diagram of the Rossler system controlled to be Lorenz system by using generalized active control; (a) shows the time series of signals x_1 and x_2, (b) shows the signal y_1 and y_2, (c) shows the signal z_1 and z_2. The arrows indicate the time we begin to control (Lorenz system: solid line, Rossler system: dash line).

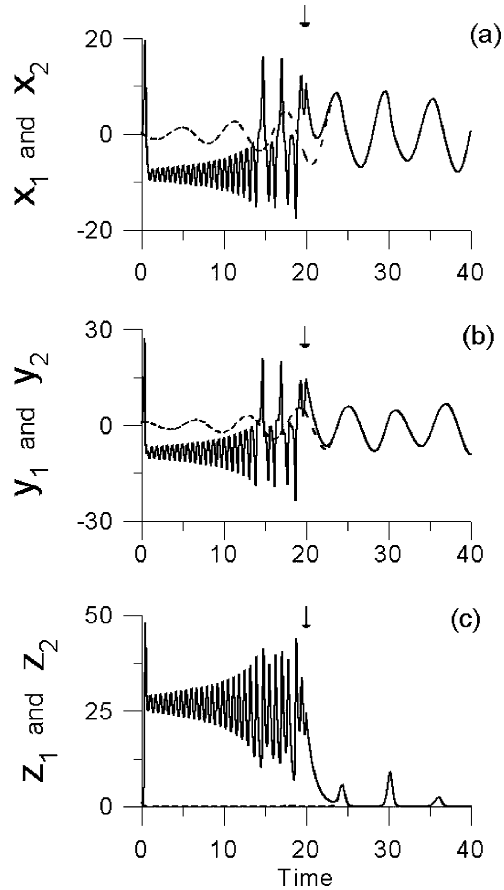

Fig. 4. The diagram of the Lorenz system controlled to be Rossler system by using generalized active control; (a) shows the time series of signals x_1 and x_2, (b) shows the signal y_1 and y_2, (c) shows the signal z_1 and z_2. The arrows indicate the time we begin to control (Lorenz system: solid line, Rossler system: dash line).

3.1. Numerical results

We select the parameters of Lorenz systems as $\sigma_1 = 10.0$, $b_1 = 2.66$, $\gamma_1 = 28$. The parameters of Rossler systems are chosen as $a_2 = 0.2$, $b_2 = 0.2$, $c_2 = 5.0$. The initial values are $x_1(0) = 0.2$, $y_1(0) = 0.6$, $z_1(0) = 1.0$ and $x_2(0) = 0.3$, $y_2(0) = 0.8$, $z_2(0) = 1.2$. Control inputs start at $t = 20$. The results of numerical simulation are presented in Fig. 3.

By the way, we can exchange master system with slave system, too. In other words, we control Lorenz system to be Rossler system. Fig. 4 presents the outcomes.

4. Noise analysis

Finally, let us explore the effect of external noise. Because the chaotic system depends sensitively on a tiny perturbation of the trajectory, we have to ensure the external noise would not destroy the synchronization. In this Letter, we focus on the example of controlling Rossler system to be Lorenz system, and rewrite (10) as

$$
\begin{aligned}
\dot{x}_1 &= \sigma_1(y_1 - x_1),\\
\dot{y}_1 &= r_1 x_1 - y_1 - x_1 z_1 + \xi(t),\\
\dot{z}_1 &= x_1 y_1 - b_1 z_1,
\end{aligned}
\qquad (15)
$$

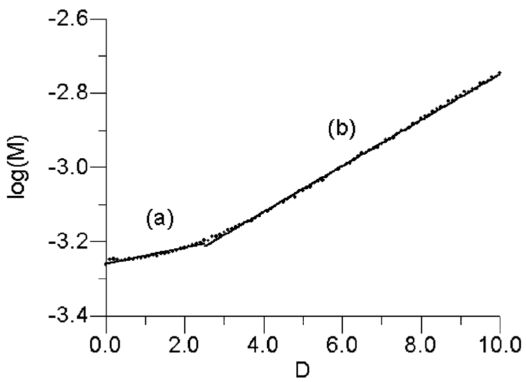

Fig. 5. The diagram of different noise density versus its responding average mean-square error M. (a) $0 \leqslant D \leqslant 2.5$, $B_1 = 0.0217224$ and $A_1 = -3.26028$; (b) $2.5 < D \leqslant 10$, $B_2 = 0.0619812$ and $A_2 = -3.36775$.

where $\xi(t)$ is the Gaussian noise satisfying $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t')$ in which D is the noise intensity. Here, $\langle \cdot \rangle$ denotes the time average. Naturally, we evaluate the mean-square error M between (x_1, y_1, z_1) and (x_2, y_2, z_2) after controlling. The definition of M is:

$$M = \frac{1}{T} \int_0^T \left[S_1(t) - S_2(t)\right]^2 dt. \tag{16}$$

Using (15) to control Rossler system, we can get various values M versus different noise intensity and observe the effect of noise.

Fig. 5 presents the results of numerical simulation. Instead of controlling the slave system after $t \geqslant 20$, we control it at the beginning. In order to diminish the effect of transient signal, we start to calculate the mean-square error after $t \geqslant 10$. In addition, when using per different noise intensity for analysis, we calculate M 500 times and average them. When $\lambda_1 = \lambda_2 = \lambda_3 = -1$, as shown in the figure, the relationship between mean-square error and noise density can be formularized as

$$M = 10^{B \times D + A}, \tag{17}$$

where

$$\begin{aligned}
&\text{(a)} \quad 0 \leqslant D \leqslant 2.5, \qquad B_1 = 0.0217224, \\
&\qquad A_1 = -3.26028, \\
&\text{(b)} \quad 2.5 < D \leqslant 10, \qquad B_2 = 0.0619812, \\
&\qquad A_2 = -3.36775.
\end{aligned} \tag{18}$$

In Eqs. (17) and (18), M becomes larger with the increase of noise density while obeying two different power laws in the range of $0 \leqslant D \leqslant 2.5$ and $2.5 \leqslant D \leqslant 10$.

5. Conclusion

This Letter demonstrates using generalized active control can synchronize two different chaos systems easily and explores the effect of external noise. We believe there still exists something interesting about these techniques.

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